

Computational Detection of Local Cadence on Revised TPS

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Abstract. Cadential Retention is a process to combine the V-I progression to regard as a single pitch event and to give a high salience, in the time-span tree of the Generative Theory of Tonal Music. Though the theory was implemented as an automatic analyzer, this process has been lacked and needed to be compensated. In this paper, we propose a fundamental procedure to realize this. We presuppose that the chord names are already assigned on the target score; then, the system first hypothesizes all the possible combinations of the key and the degree, finds plausible connections between them, and detects cadences. We employ dynamic programming to connect chords, and calculate the chord distance by the revised Tonal Pitch Space. Also, we restrict the minor scale to the harmonic one to avoid the ambiguity in chord interpretation. In our system, we show the final result of the revised time-span tree, including the local cadences.

Keywords: Cadence, Generative Theory of Tonal Music, the Tonal Pitch Space

1 Introduction

The Generative Theory of Tonal Music (GTTM), proposed by Lerdahl and Jackendoff [1] is one of the most promising theories of constructive musicology, which retrieves the latent structure of music. This theory consists of comparatively rigorous rules for analysis though the application of the rules is sometimes ambiguous. Hamanaka *et al.* [2–4] have implemented the theory on computer, and has opened the analyzer exGTTM as free use, however, it has not included *cadential retention* that is to identify the cadences and to give high saliency in the time-span tree.

In finding cadences, we need the preprocess of chord analysis, that is to identify the key and the degree for each chord. Furthermore, in order to acquire the chord information, we need to distinguish the constituent notes in a chord from those passing or auxiliary notes, suspensions, or appoggiatura, and also in some cases we may need to compensate the missing roots.

On the other hand, the recent music scores, especially in popular music, accompany those Berklee chord names in the style called *lead sheet*. This kind

of sheet music is helpful for us to identify chord functions, however, we may still need the information on the distance between chords, and thus we employ the Tonal Pitch Space (TPS) [5].

In this paper, we first show the process to find cadences, including so-called *local cadences*, that work as local articulation in a music piece. Then, we introduce Viterbi algorithm to find plausible connections of chords with the revised TPS on the harmonic minor scale. Lastly, we present the whole process to compose the time-span tree with cadential retention.

This paper is organized as follows. In related works in Section 2, we introduce the preceded works which directly concern our current research, as well as fundamental music theories. In Section 3, we give our formal representation and algorithm on finding local cadences. In Section 4, we show our experimental results and in Section 5 we summarize our contribution.

2 Related Works

2.1 Cadence in GTTM

GTTM first articulates the music score into *groups*; the shorter ones correspond to phrases/motifs and the combined longer group results in a whole melody. Thereafter, the theory assigns the metrical importance to each pitch event based on its rhythm. Regarding those pitch events locating at the both ends of each group and/or those with the metrically strong beats more salient, GTTM composes the *time-span* tree where the salient events extend more upwards, compared with neighboring events (branches), absorbing those subordinate branches. Finally, the time-span tree is reorganized to the *prolongational* tree including the chord information, which represents the tonal stability.

Although we can disregard the chord progression in the time-span analysis, the *cadence* is the exception; we need to connect the two chords of V-I and to regard as a single pitch event. In the original theory [1], the process of chord analysis was not mentioned, and thus, the implemented time-span analyzer [2] has also lacked this process.

Later, Lerdahl presented the compensating theory called Tonal Pitch Space (TPS) [5] to give a clue to identify the chord functions. TPS calculates the distance of two given chords. The progression from V to I, for example, is quite smooth, and thus V-I has a shorter distance. Within a key, the less natural progression owns the larger value of distance. Also, the progression to a chord in a modulation owns a longer distance. Among the modulations, the migration to the unrelated key from the current key has further penalty. In this way, the theory finds the shortest distance among chords, and identifies their chord functions as well as the dominant motion.

2.2 Harmony Analysis with TPS

Sakamoto [6] has proposed a Viterbi algorithm, given a network of candidates of chord identifiers, to find a path with the shortest distance based on the Tonal Pitch Space. This process is detailed as follows.

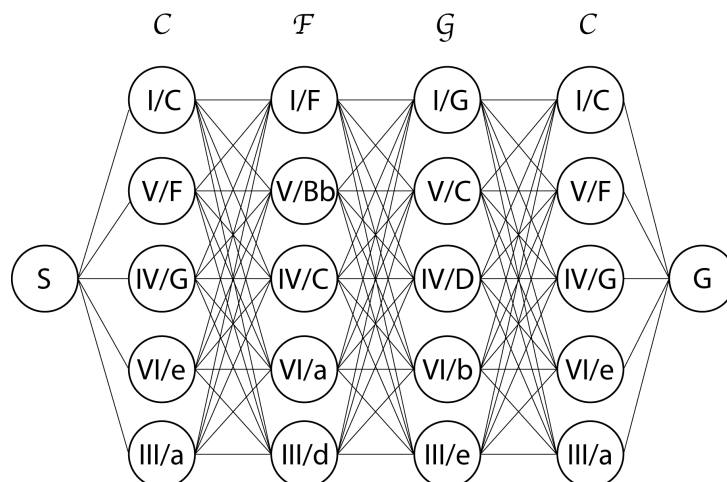


Fig. 1. Graphic representation of candidate harmony

1. Given a sequence of Berklee chord names; *e.g.*, $C \Rightarrow F \Rightarrow G \Rightarrow C$.
2. Assign possible combinations for each chord name; *e.g.*, C can be regarded either I/C , IV/G , V/F , VI/e or III/a (Roman numerals denote the degrees of the chord and bold alphabets denote the keys).
3. A pair of adjacent chord names produces a bipartite graph since the chord names are interpreted in multiple ways and are connected by possible links as shown in Fig. 1.
4. Calculate the distance by the TPS from the start node to the goal node by Viterbi dynamic programming; *e.g.*, in case $C \Rightarrow F \Rightarrow G \Rightarrow C$ is given, the path of $I/C - IV/C - V/C - I/C$ becomes the shortest.

We regard that the numerically smallest path, *i.e.*, the shortest path is the most plausible interpretation of keys and degrees. This intuition is supported by the following criteria.

- In general, the distance between two chords becomes smaller when they do not include modulation, and the modulation does not occur unnecessarily.
- The cadential progression such as $V-I$ becomes shorter distance, and thus the shorter distance reflects the cadential progression which is more smooth in motion.

2.3 The Revised Tonal Pitch Space

Yamaguchi *et al.*[7] has revised the theory of Tonal Pitch Space to analyze jazz. They claimed that the defects of the original theory are (i) the variety of chords was too limited to treat modern popular music, and (ii) those non-diatonic chords which often appear in the current jazz should be included in the analysis. The

level <i>a</i> (root):	0											
level <i>b</i> (fifth):	0				7							
level <i>c</i> (triadic):	0		4		7							
level <i>d</i> (chordal):	0		4		7				10			
level <i>e</i> (diatonic):	0	2	4	5	7	9	10	11				
level <i>f</i> (chromatic):	0	1	2	3	4	5	6	7	8	9	10	11

Fig. 2. Basic space of I_7/C based on Yamaguchi’s extension. They added another level called *chordal* between triadic and diatonic level.

main issue here was that extended constitute notes such as the seventh (7-th) tone of tetrad chord was classified at the level *c* (triadic) and gave the same value as the third; the seventh is often omitted whereas the third owes a critical role to decide whether major or minor. They have revised the theory to add another level called *chordal*, to distinguish the third and the seventh, between level *c* (triadic) and *e* (diatonic) as shown in Fig. 2.

2.4 Revision on Cadential Retention

Matsubara *et al.*[8] revised the process of cadential retention, and added the half cadence by the secondary dominant (Doppel-dominant) and the local cadence. As for the latter, the notion is ill-defined in the previous work and thus we redefine it in the following section. The half cadence is mainly organized solely by a single V (dominant) chord, however, it often appears following the double-dominant as $V/V-V/I$ or $V/V-V/i$. Since this progression can be regarded in the similar way to $V-I$ in the full cadence, Matsubara formalized the process to unify the two events as one, like other cadential retention, proposing the *half-egg*-symbol as opposed to *egg*-symbol in the full cadence in GTTM.

They proposed the algorithm of cadential retention as follows.

1. Group such two pitch events e_1 , e_2 as $V-I$, $VI-I$, or $V/V-V/I$, and assign functions according to Viterbi algorithm [6]. Here, we disregard the following two consecutive pitch events.
 - when two ends of the pitch events do not coincide.
 - when the end of unified two events does not coincide with the end of the upper group.
2. Unify e_1 and e_2 as one pitch event, assigning the *egg*-marker and regarding the head of the current group. Treat this head as a single event in the upper group.
3. Repeat the process by the root of the whole tree.

3 Formal Representation of Local Cadence

We propose the revised TPS, integrating Yamaguchi[7] and Matsubara[8].

- We restrict the minor scale to the harmonic one.
- We reinterpret the chord which has a dominant function (Fig. 3).
- We add another chordal level to the basic space.

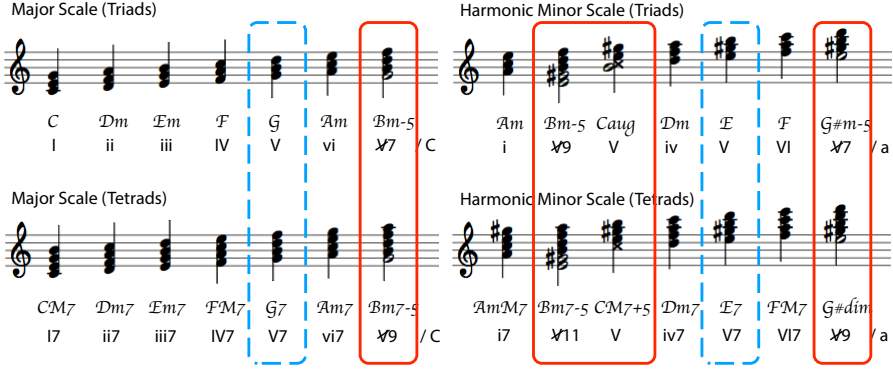


Fig. 3. Restricting the harmonic minor scale and reinterpreting the chord which has a dominant function. The chords in the broken lines are dominants and those in the solid lines are extended.

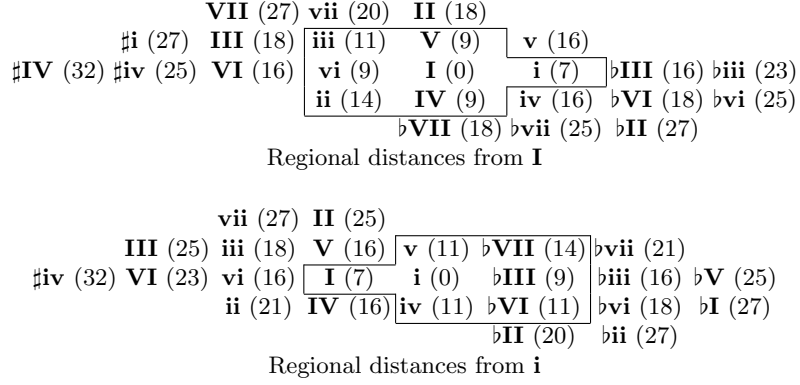


Fig. 4. Regional space of revised TPS

3.1 Redefine Regional Distance of TPS

By restricting to the harmonic minor scale and adding the level to the basic space, we need to recalculate the basic space distance $\text{basic_space}(x, y)$ between the chords.

Here, we describe the regional distance $\Delta(R_x, R_y)$ between the tonic chords of different keys. Fig. 4 shows the regional space of the revised TPS. The part surrounded by the convex polygon is the related keys. The distance increases in general as compared with the conventional one. However, since the regional distances does not increase uniformly; in minor keys, the shortest path from **i** to **ii** and **bvii** decreased.

The comparison with the human musical intuition of the regional distance redefined by the proposed method is a future work.

Table 1. Types of chords, corresponding Berklee chord names, and a list of interpretations

Chord type	Symbol	Interpretation with	
		TPS	Revised TPS
Major triad	C	<u>I/I</u> , III/vi, IV/V, V/IV, VI/iii, VII/ii	<u>I/I</u> , III/vi, IV/V, V/IV, V/iv
Minor triad	Cm	i/i, ii/bVII, iii/bVI, iv/v, v/iv, vi/bIII	i/i, ii/bVII, iii/bVI, iv/v, vi/bIII
Diminished triad	Cdim	ii°/bvii, vii°/bII	ii°/bvii, <u>X₉/bvii</u> , vii°/bII, <u>X₇/bII</u> , vii°/♯i, <u>X₇/♯i</u>
Augmented triad	Caug	(N/A)	III ⁺ /♯i, V/♯i, III ⁺ /iv, V/iv, III ⁺ /vi, V/vi
Dominant seventh	C7	V ₇ /IV, VII ₇ /ii	V ₇ /IV, V ₇ /iv
Major seventh	CM7	<u>I₇/I</u> , III ₇ /vi, IV ₇ /V, VI ₇ /iii	<u>I₇/I</u> , IV ₇ /V, VI ₇ /iii
Minor seventh	Cm7	i ₇ /i, ii ₇ /bVII, iii ₇ /bVI, iv ₇ /v, v ₇ /iv, vi ₇ /bIII	ii ₇ /bVII, iii ₇ /bVI, iv ₇ /v, vi ₇ /bIII
Diminished seventh	Cdim7	(N/A)	vii ₇ °/♯i, <u>X₉/♯i</u> , vii ₇ °/iii, <u>X₉/iii</u> , vii ₇ °/v, <u>X₉/v</u> , vii ₇ °/bvii, <u>X₉/bvii</u>
Augmented seventh	CM7+5	(N/A)	III ₇ ⁺ /vi, V/vi
Half diminished seventh	C∅	ii ₇ [∅] /bvii, vii ₇ [∅] /bII	ii ₇ [∅] /bvii, <u>X₁₁/bvii</u> vii ₇ [∅] /bII, <u>X₉/bII</u>
Minor major seventh	CmM7	(N/A)	<u>i₇/i</u>
Major ninth	C9	V ₉ /IV	V ₉ /IV
Minor ninth	C7-9	VII ₉ /ii	V ₉ /iv

3.2 Reinterpretation of dominant function by revised TPS

By restricting to the scale and reinterpreting the dominant chords, possible interpretations of Berklee chord name increase. Table 1 shows types of chords and corresponding Berklee chord name and a list of interpretations before and after applying the proposed method. Here, the root is C in the chord symbol column, and C major is described as I in the interpretation column. The one underlined represents what has been interpreted as a chord having a dominant function by correcting the constituent note.

3.3 Local Cadence Formalization

Extending half cadence In this research, we formalize the extension of cadence proposed by Matsubara [8]. We redefine the all dominant chords on the V with tetrad or pentad with / without root as the secondary chord of half cadence. Table 2 shows all the harmonic progression to be a cadence that made the above extension. The chord candidates with dominant function is limited to

Table 2. Extending dominant function: Where, D is a dominant (any function whose degree is V , including ones with missing root).

Kind of cadence	Major key	Minor key
Full cadence	$V/\mathbf{I} \rightarrow \mathbf{I}/\mathbf{I}$	$V/\mathbf{i} \rightarrow \mathbf{i}/\mathbf{i}$
	$V_7/\mathbf{I} \rightarrow \mathbf{I}/\mathbf{I}$	$V_7/\mathbf{i} \rightarrow \mathbf{i}/\mathbf{i}$
Deceptive cadence	$V/\mathbf{I} \rightarrow \mathbf{vi}/\mathbf{I}$	$V/\mathbf{i} \rightarrow \mathbf{VI}/\mathbf{i}$
	$V_7/\mathbf{I} \rightarrow \mathbf{vi}/\mathbf{I}$	$V_7/\mathbf{i} \rightarrow \mathbf{VI}/\mathbf{i}$
Half cadence (1 chord)	$\rightarrow V/\mathbf{I}$	$\rightarrow V/\mathbf{i}$
Half cadence (2 chords)	$D/\mathbf{V} \rightarrow V/\mathbf{I}$	$D/\mathbf{v} \rightarrow V/\mathbf{i}$
	$(D \in \{V, V_7, \mathcal{X}_7, V_9, \mathcal{X}_9, V_{11}, \mathcal{X}_{11}\})$	

the secondary dominant which precedes the half cadence in this research. This is because dominant chords of the full cadence or the deceptive cadence are limited to V and V_7 in the classical era, and other chords on V will not be treated as cadence at the end of harmony progression.

Local cadence Consider the chord progression $c_{i-1} \rightarrow c_i$, which is not a cadence, but to be a “local cadence” if c_{i-1} and c_i can satisfy the conditions by selecting their harmonic functions arbitrarily.

This definition is aimed at carrying out cadential retention to the harmony progression which is not to be a cadence as a whole, but to be a cadence if ignoring the context before and after its appearance. As an example of local cadence, Fig. 5 shows the harmony analysis of R. Wagner’s “Tannhäuser Overture”. According to the analysis, the chord progression $\mathbf{E}\flat \rightarrow \mathbf{F}\mathbf{m}$ in the bars 3 – 4 was interpreted as $\mathbf{I}/\mathbf{E}\flat \rightarrow \mathbf{ii}/\mathbf{E}\flat$, and another possible interpretation $V/\mathbf{A}\flat \rightarrow \mathbf{vi}/\mathbf{A}\flat$ was not selected. Ignoring the contexts before and after it, these chords can be interpreted as chords on $\mathbf{A}\flat$, thus we can treat this as the deceptive cadence.

3.4 Computational implementation

Our proposed method consists of the following three steps: Harmony analysis, Cadence detection, and Cadential retention.

1. **Harmony analysis:** Given a MusicXML of monophonic melody with Berklee chord names, outputs Harmony Graph which illustrates the most plausible interpretation of chord progressions by Viterbi algorithm described in Section 2.2.
2. **Cadence detection:** Combining the chord progressions, GroupingXML and Time-spanXML derived from exGTTM[9], detects the progression to be a cadence including a local cadence.
3. **Cadential retention:** Assigning the cadential retention to the detected cadence, rearranges the Time-span tree, and outputs Time-spanXML and Time-span tree visualization.

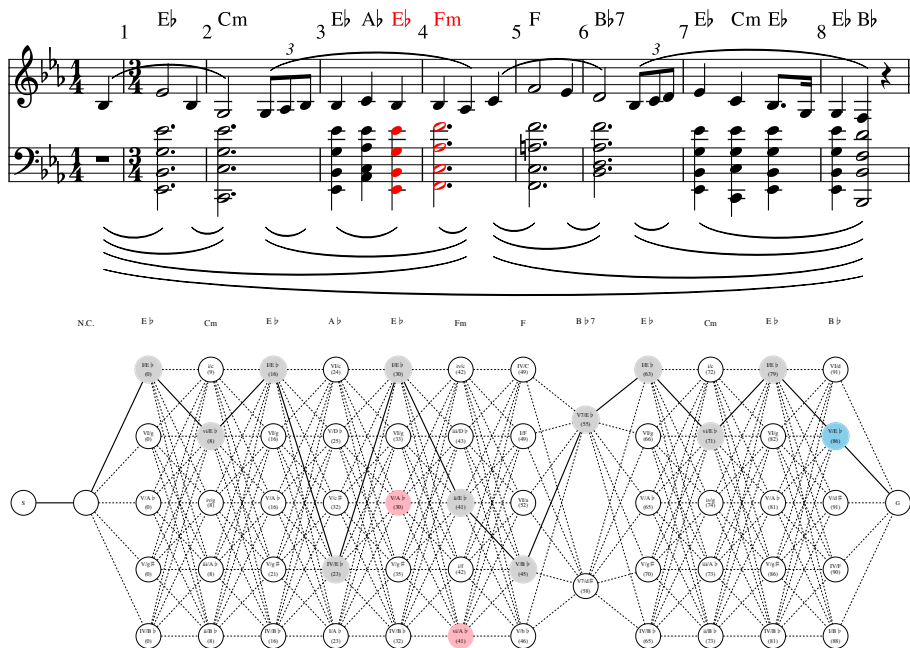


Fig. 5. Example of detecting local cadence in harmony graph. (Upper) Score of R. Wagner’s “Tannhäuser Overture” with Berklee chord names and grouping structure. (Lower) Harmony graph. The red symbols and circles indicate local cadence and the blue symbol indicates half or full cadence.

4 Experimental Results and Discussions

We used GroupingXML, Time-spanXML from GTTM database[10], and MusicXML files as input data. We selected 127 out of 300 phrases whose Berklee chord names were printed in the score[11]. We introduce representative results and discuss them.

Harmony analysis Fig. 6 shows the experimental results of harmony analysis at the beginning of F. Chopin’s “The Preludes, op. 28–15”. In the harmony graph, the solid lines indicate estimated chord progression and blue nodes represent the normal cadence, and red nodes are the local cadence.

From the result of harmony analysis, this phrase consisted of only $V_7/D\flat$ and $I/D\flat$, namely all $A\flat_7$ to $D\flat$ were to be a progression of full cadence. However, integrating the results of harmony, grouping, and time-span analysis, our proposed method can detect the cadence progressions.

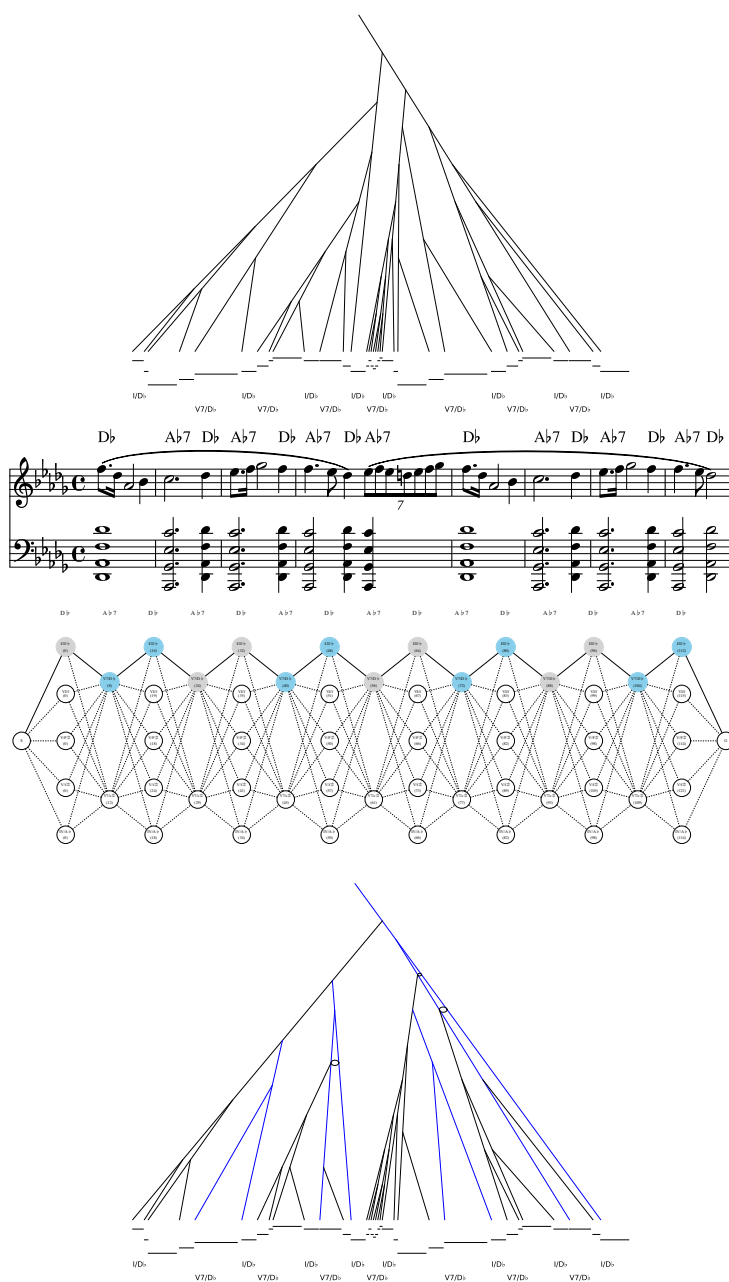


Fig. 6. Results of F. Chopin’s “The Preludes, op. 28–15”. (Upper) Original Time-span tree from GTTM database and score with Berklee chord names as input data. (Lower) Harmony graph and Time-span tree with cadential retention as outputs. The blue branches were detected as the cadence and rearranged.

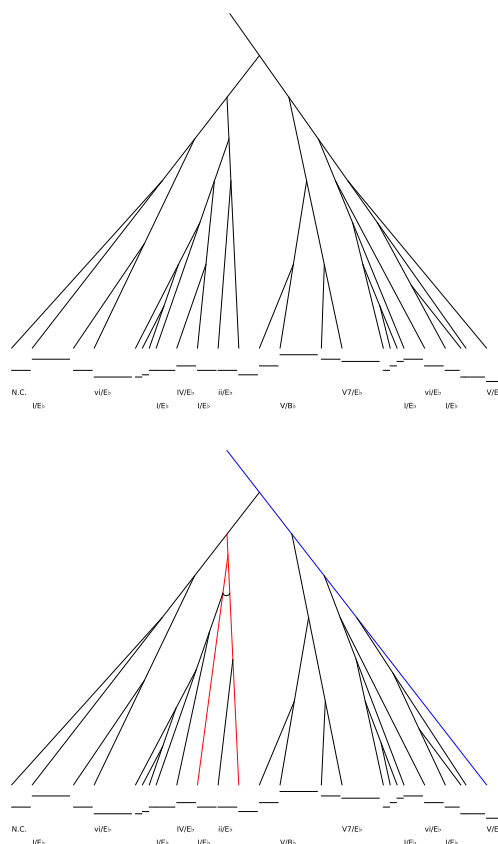


Fig. 7. Result of R. Wagner’s “Tannhäuser Overture”. (Upper) Original Time-span tree from GTTM database. (Lower) Time-span tree with cadential retention. The blue branches were detected as the full cadence and the red ones were detected as the local cadence, and they were rearranged. Score and harmony graph were shown in Fig. 5.

Cadential Retention Fig. 7 shows the results of cadential retention at the beginning of R. Wagner’s “Tannhäuser Overture”. As shown in the bars 3–4 in Fig. 5, a possible interpretation $V/A^b \rightarrow vi/A^b$ chords can be interpreted as chords on A^b as local deceptive cadence. After detecting the local cadence, Time-span tree was modified at the third beat of bars 3 from where the penult of local cadence begins. As a result, the penult and final were unified at higher level in the Time-span tree.

5 Conclusion

In this research, we showed our subprocess of cadential retention, which has been left unaccomplished in the automatic analyzer of GTTM. We presupposed

that the Berklee chord names were already assigned on the target score, and restricted the minor scale to the harmonic one to avoid the ambiguity in chord interpretation.

Then, the system first hypothesized all the possible combinations of the keys and the degrees, looked for plausible connections between them, and identified the path with the shortest distance where we employed Viterbi algorithm with the revised TPS. We have shown several results of cadential retention, as well as local cadences, in the tree format.

Our contribution is not only the compensation for the GTTM analyzer. In addition to the use of Viterbi algorithm and the revised TPS, we clarified the articulation by local cadences, which has not been included in the original theory.

The original contribution of GTTM is to externalize the hidden rules in music composition. Then, we have added such rules as cadential retention, including the local ones, to the original theory. We believe that the formulation of rules would contribute to the music creativity, since we could evoke these rules in the generative process of music.

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References

1. Lerdahl, F. and Jackendoff, R.: A Generative Theory of Tonal Music. The MIT Press (1983)
2. Hamanaka, M., Hirata, K. and Tojo, S.: Implementing A Generative Theory of Tonal Music. *Journal of New Music Research*, 35(4), pp. 249–277 (2007)
3. Hamanaka, M., Hirata, K. and Tojo, S.: Implementing Methods for Analysing Music Based on Lerdahl and Jackendoff’s Generative Theory of Tonal Music. In: Meredith, D. (eds.) *Computational Music Analysis*, Chapter 9, pp. 221–249, Springer (2016)
4. Hirata, K., Tojo, S. and Hamanaka, M.: An Algebraic Approach to Time-Span Reduction. In: Meredith, D. (eds.) *Computational Music Analysis*, Chapter 10, pp. 251–270, Springer (2016)
5. Lerdahl, F.: *Tonal Pitch Space*. Oxford University Press (2001)
6. Sakamoto, S., Arn, S., Matsubara, M. and Tojo, S.: Harmonic analysis based on Tonal Pitch Space. In: proceedings of the 8th International Conference on Knowledge and Systems Engineering (KSE), pp. 230–233 (2016)
7. Yamaguchi, N. and Sugamura, N.: Improving TPS to Tackle Non Key Constituent Note. *IPSJ SIG Technical Report*, Vol.2011-MUS-89, No.10, 6 pages, (2011) (in Japanese)
8. Matsubara, M., Kodama, T. and Tojo, S.: Revisiting Cadential Retention in GTTM. In: proceedings of the 8th International Conference on Knowledge and Systems Engineering (KSE), pp. 218–223 (2016)
9. Hamanaka, M., Hirata, K. and Tojo, S.: ATTA: Automatic Time-span Tree Analyzer Based on Extended GTTM. In: Proceedings of the 6th International Conference on Music Information Retrieval (ISMIR), pp. 358–365 (2005)

10. Hamanaka, M., Hirata, K. and Tojo, S.: Musical structural analysis database based on gttm. In: Proceedings of the 15th International Conference on Music Information Retrieval (ISMIR) 2014, pp. 325–330 (2014)
11. Matsuyama, Y.: Classic Melody File. DoReMi Music publishing (2011)